Logics of Truthmaker Semantics: Comparison, Compactness and Decidability

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Extract from MSc thesis, supervised by Johan van Benthem and Nick Bezhanishvili
PhD Supervisors: Maria Aloni and Nick Bezhanishvili

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Universiteit van Amsterdam

Outline of the talk

- Motivation and general aim
- Defining the truthmaker framework
- Presenting proof (outlines) of formal properties of ‘truthmaker logics’.
- Conclusion and current research

Motivation and general aim

Background

- Truthmaker semantics (TS) was introduced to model ‘exact truthmaking’.
- Great interest in TS as a framework for analyzing various philosophical and linguistic phenomena, e.g., metaphysical grounding, counterfactuals and implicatures [cf. Fine (2017)].
- But limited study of the various logics arising from the semantics [exception being Fine and Jago (2019)].

This talk aims to address this gap by exploring numerous ‘truthmaker logics’

In particular:

1. Translations and Compactness
2. Finite Model Property (FMP) and Decidability
3. Connection with modal (informational) logic [will probably be skipped]

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Defining truthmaker semantics

Definition (language and semantics)

The language is given by

$$\varphi ::= \; p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi.$$

The semantics are *bilateral* (truthmaking $\Vdash^+$ and falsitymaking $\Vdash^-$), and models come with two valuations V^+, V^-:

$$\begin{aligned}
\mathbb{M}, s \Vdash^{\pm} p \quad & \text{iff} \quad s \in V^{\pm}(p). \\
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& \qquad s = \sup\{t, t'\})
\end{aligned}$$

Example

How about '∨' and falsitymaking '∧'?

Truthmaker framework: *Semantics* parameter

Non-incl.: $\mathbb{M}, s \Vdash^+ \varphi \vee \psi$ iff $\mathbb{M}, s \Vdash^+ \varphi$ *or* $\mathbb{M}, s \Vdash^+ \psi$.

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Falsitymaking $\Vdash^-$ of '∨' and '∧' is defined by de Morgan's.

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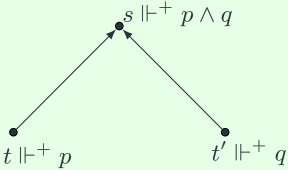

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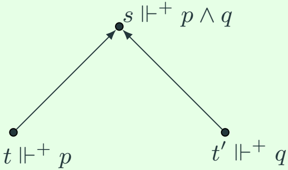

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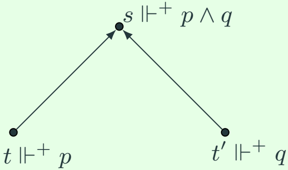

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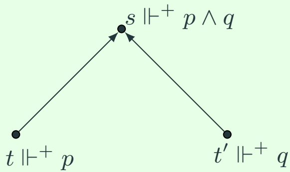

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Truthmaker framework: *Valuation* parameter

- **All:** Any pairs of valuations $V^{\pm} : \mathbf{P} \to \mathcal{P}(S)$ are admissible.
- **Closure under binary joins:** if $\{s, t\} \subseteq V^{\pm}(p)$, then $\sup\{s, t\} \in V^{\pm}(p)$.
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$$\mathcal{S}_1 := \{(S, \leq) \mid (S, \leq) \textit{ is a semilattice}\}.$$

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(so $\mathcal{C}_2$ is the class of complete lattices.)

Truthmaker logics

For any choice of semantics, valuations and frames, we get a *truthmaker consequence relation* by defining

$$\Gamma \Vdash^{+} \varphi \text{ :iff whenever } \mathbb{M}, s \Vdash^{+} \gamma \text{ for all } \gamma \in \Gamma\text{, it is also the case that } \mathbb{M}, s \Vdash^{+} \varphi$$

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So we have (at least) $2 \times 4 \times 4 = 32$ logics to survey $\ldots$

Luckily, they can be dealt with (rather) uniformly!

Proof strategy

Proceeding from here, our proof strategy is as follows:

1. Inherit compactness and recursive enumerability from first-order logic through translations *for semilattice truthmaker logics*.
2. Develop and prove a truthmaker analogue of the finite model property to obtain decidability *for semilattice truthmaker logics*.
3. Show that truthmaker consequence is invariant for choice of frames,which also entails that 'all' truthmaker logics are (compact and) decidable.

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Translations into first-order logic

Definition (translation into FOL)

We define the following translation-pair into first-order logic (FOL):

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\begin{aligned}
ST^+_x(p) &= P^T x \\
ST^-_x(p) &= P^F x \\
ST^+_x(\neg\phi) &= ST^-_x(\phi) \\
ST^-_x(\neg\phi) &= ST^+_x(\phi) \\
ST^+_x(\phi \wedge \psi) &= \exists y, z\big(x = \sup\{y, z\} \wedge ST^+_y(\phi) \wedge ST^+_z(\psi)\big) \\
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Proposition (correspondence)

For all models $\mathbb{M}$ and all $\varphi \in \mathcal{L}_T$, we have:

For all states $s \in \mathbb{M}$: (i) $\mathbb{M}, s \Vdash^+ \varphi$ iff $\mathbb{M} \vDash ST^+_x(\varphi)[s]$; *and*

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Compactness and recursive enumerability

Proposition (semilattice compactness and r.e.)

All semilattice truthmaker logics are

- **compact:** *if* $\Gamma \Vdash^+ \varphi$, *then* $\Gamma_F \Vdash^+ \varphi$ for some finite $\Gamma_F \subseteq \Gamma$; and
- **r.e.:** For finite Γ_F, we can effectively enumerate (Γ_F, φ) s.t. $\Gamma_F \Vdash^+ \varphi$.

Proof.

Let J be the first-order formula defining (join)-semilattices. For **compactness**, the argument is essentially that

$$\begin{aligned} \Gamma \Vdash^+ \varphi \quad &\text{iff} \quad ST_x^+(\Gamma) \cup \{J\} \vDash ST_x^+(\varphi) \\ &\overset{(c)}{\text{iff}} \quad ST_x^+(\Gamma_F) \cup \{J\} \vDash ST_x^+(\varphi) \quad \text{iff} \quad \Gamma_F \Vdash^+ \varphi, \end{aligned}$$

where $\Gamma_F \subseteq \Gamma$ is finite.
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$$\Gamma \Vdash^+ \varphi \quad \text{iff} \quad ST^+_x(\Gamma) \cup \{J\} \vDash ST^+_x(\varphi)$$
$$\overset{(c)}{\text{iff}} \quad ST^+_x(\Gamma_F) \cup \{J\} \vDash ST^+_x(\varphi) \quad \text{iff} \quad \Gamma_F \Vdash^+ \varphi,$$

where $\Gamma_F \subseteq \Gamma$ is finite.

For **r.e.**, the argument is essentially that

$$\Gamma_F \Vdash^+ \varphi \quad \text{iff} \quad ST^+_x(\Gamma_F) \cup \{J\} \vDash ST^+_x(\varphi)$$
$$\text{iff} \quad \vDash \bigwedge (ST^+_x(\Gamma_F) \cup \{J\}) \to ST^+_x(\varphi).$$

□

Compactness and recursive enumerability

Proposition (semilattice compactness and r.e.)

All semilattice truthmaker logics are

- **compact:** *if* $\Gamma \Vdash^{+} \varphi$, *then* $\Gamma_F \Vdash^{+} \varphi$ for some finite $\Gamma_F \subseteq \Gamma$; and
- **r.e.:** For finite Γ_F, we can effectively enumerate (Γ_F, φ) s.t. $\Gamma_F \Vdash^{+} \varphi$.

Proof.

Let J be the first-order formula defining (join)-semilattices. For **compactness**, the argument is essentially that

$$\begin{array}{llll} \Gamma \Vdash^{+} \varphi & \text{iff} & ST_x^{+}(\Gamma) \cup \{J\} \vDash ST_x^{+}(\varphi) & \\ & \overset{(c)}{\text{iff}} & ST_x^{+}(\Gamma_F) \cup \{J\} \vDash ST_x^{+}(\varphi) & \text{iff} \quad \Gamma_F \Vdash^{+} \varphi, \end{array}$$

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FMP and decidability

Initial observation: A direct analogue of, e.g., the modal logic FMP is trivial and does nothing for proving decidability. Instead, we prove the following:

Theorem (Truthmaker FMP)

For any model $\mathbb{M}_0 = (S_0, \leq_0, V_0^+, V_0^-)$, state $s \in S_0$, and finite set of formulas $\Gamma_F \subseteq \mathcal{L}_T$ s.t. $\mathbb{M}_0, s \Vdash^+ \Gamma_F$,
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What about other classes of frames?

Limitation of translation method: it only applies when conditions are first-order definable. And having, e.g., all joins is not.

Second-order frames

Definition (recall)

$$\mathcal{S}_1 := \{(S, \leq) \mid (S, \leq) \textit{ is a semilattice}\},$$
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Theorem (Entailment Invariance for Choice of Frames)

Given any choice of semantics and valuations, and any $X, Y \in \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{C}_1, \mathcal{C}_2\}$,

$$\Gamma \Vdash^+_X \varphi \quad \text{iff} \quad \Gamma \Vdash^+_Y \varphi.$$

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Clearly, $\Gamma \Vdash^+_{\mathcal{S}_1} \varphi \Rightarrow \Gamma \Vdash^+_{\mathcal{S}_2/\mathcal{C}_1} \varphi \Rightarrow \Gamma \Vdash^+_{\mathcal{C}_2} \varphi.$

Therefore, $\Gamma \Vdash^+_{\mathcal{S}_1} \varphi \Leftarrow \Gamma \Vdash^+_{\mathcal{C}_2} \varphi$ suffices, which is a consequence of our Completion Lemma showing how to complete a semilattice into a complete lattice in a satisfaction-preserving and -reflecting way. □

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Summary:

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- Developed and proved the FMP, achieving decidability (for some truthmaker logics).
- Showed that truthmaker consequence is invariant for choice of frames, allowing us to additionally conclude that ‘all’ truthmaker logics are (compact and) decidable.
- Gave a modal perspective on truthmaker semantics: adding classical negation results in modal information logic.

Current research (interests):

- In particular, working on developing a modal perspective on team semantics.
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- In particular, working on developing a modal perspective on team semantics.
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Conclusion and current research

Summary:

- Translated into FOL, achieving r.e. and compactness (for some truthmaker logics).
- Developed and proved the FMP, achieving decidability (for some truthmaker logics).
- Showed that truthmaker consequence is invariant for choice of frames, allowing us to additionally conclude that 'all' truthmaker logics are (compact and) decidable.
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Thank you!

References I

Fine, K. (2017). "Truthmaker Semantics". In: *A Companion to the Philosophy of Language*. John Wiley Sons, Ltd. Chap. 22, pp. 556–577. DOI: `https://doi.org/10.1002/9781118972090.ch22` (cit. on pp. 3–7).

Fine, K. and M. Jago (2019). "Logic for Exact Entailment". In: *The Review of Symbolic Logic* 12.3, pp. 536–556. DOI: `10.1017/S1755020318000151` (cit. on pp. 3–7).

Van Benthem, J. (2019). "Implicit and Explicit Stances in Logic". In: *Journal of Philosophical Logic* 48.3, pp. 571–601. DOI: `10.1007/s10992-018-9485-y` (cit. on p. 61).

Completion Lemma

Lemma

Let $(S, \leq)$ be a semilattice and $\mathcal{U}(S) \subseteq \mathcal{P}(S)$ its collection of upsets. Then (i) $(\mathcal{U}(S), \supseteq)$ forms a complete lattice, and (ii) for all $s, t, u \in S$:

$$s = \sup_{\leq}\{t, u\} \qquad \text{iff} \qquad {\uparrow}s = {\uparrow}t \cap {\uparrow}u.$$

Lemma

For all formulas $\varphi \in \mathcal{L}_T$ and $\mathbb{M}, s$ s.t. $\mathbb{M}, s \Vdash^+ \varphi$, there are literals $l_1, \ldots l_n$ s.t.

1. $(l_1 \wedge \cdots \wedge l_n) \Vdash^+_{\mathcal{S}_1} \varphi$,
2. $\mathbb{M}, s \Vdash^+ (l_1 \wedge \cdots \wedge l_n)$.

Completion Lemma

Let $\mathbb{M} = (S, \leq, V^+, V^-)$ be a semilattice model. Then for all $\varphi \in \mathcal{L}_T$ and all $s \in S$,

$$\left(\mathcal{U}(S), \supseteq, V'^+, V'^-\right), {\uparrow}s \Vdash^+ \varphi \qquad \text{iff} \qquad \mathbb{M}, s \Vdash^+ \varphi,$$

where $V'^{\pm}(p) := \{{\uparrow}s \mid s \in V^{\pm}(p)\}$.

A modal perspective on truthmaker semantics

Definition (van Benthem (2019)'s translation)

Let $\mathcal{L}_M$ be the language of modal information logic; i.e., the modal language with a single binary modality '$\langle \mathrm{sup} \rangle$' (for supremum). Define the following translation:

$$
\begin{aligned}
(p)^+ &= p^T, & (p)^- &= p^F, \\
(\neg\varphi)^+ &= \varphi^-, & (\neg\varphi)^- &= \varphi^+, \\
(\varphi \wedge \psi)^+ &= \langle \mathrm{sup} \rangle \varphi^+ \psi^+, & (\varphi \wedge \psi)^- &= \varphi^- \vee \psi^-, \\
(\varphi \vee \psi)^+ &= \varphi^+ \vee \psi^+, & (\varphi \vee \psi)^- &= \langle \mathrm{sup} \rangle \varphi^- \psi^-.
\end{aligned}
$$

Definition

Let $\mathcal{L}_M^{\{p^T, p^F, \vee, \langle \mathrm{sup} \rangle\}} \subseteq \mathcal{L}_M$ be the fragment of the language of modal information logic restricted to the propositional letters, connective '$\vee$' and modality '$\langle \mathrm{sup} \rangle$'. Define the following translation:

$$
\begin{aligned}
(p^T)^\bullet &= p, & (p^F)^\bullet &= \neg p, \\
(\langle \mathrm{sup} \rangle \varphi\psi)^\bullet &= \varphi^\bullet \wedge \psi^\bullet, & (\varphi \vee \psi)^\bullet &= \varphi^\bullet \vee \psi^\bullet.
\end{aligned}
$$

A modal perspective on truthmaker semantics (continued)

Proposition

The translations $(\cdot)^+$ and $(\cdot)^\bullet$ are each other's 'inverses':

For all $\varphi \in \mathcal{L}_T$ *and all* $\mathbb{M}, s$*:* $\mathbb{M}, s \Vdash^+ \varphi$ **iff** $\mathbb{M}, s \Vdash^+ (\varphi^+)^\bullet$.

For all $\varphi \in \mathcal{L}_M^{\{p^T, p^F, \vee, \langle\sup\rangle\}}$ *and all* $\mathbb{M}, s$*:* $\mathbb{M}, s \Vdash \varphi$ **iff** $\mathbb{M}, s \Vdash (\varphi^\bullet)^+$.

Corollary (Characterization)

Truthmaker logics are (in a precise mathematical sense) the $\{\vee, \langle\sup\rangle\}$-fragments of modal information logics, or alternatively, modal information logics arise from augmenting truthmaker logics with classical negation.